

Calculus 12
Trigonometry Review

A. Find the exact value of each ratio. Do not use a calculator.

1) $\sin 0$ 2) $\sin \frac{\pi}{6}$ 3) $\sin \frac{\pi}{4}$ 4) $\sin \frac{\pi}{3}$ 5) $\sin \frac{\pi}{2}$

6) $\cos 0$ 7) $\cos \frac{\pi}{6}$ 8) $\cos \frac{\pi}{4}$ 9) $\cos \frac{\pi}{3}$ 10) $\cos \frac{\pi}{2}$

11) $\tan 0$ 12) $\tan \frac{\pi}{6}$ 13) $\tan \frac{\pi}{4}$ 14) $\tan \frac{\pi}{3}$ 15) $\tan \frac{\pi}{2}$

16) $\csc \frac{2\pi}{3}$ 17) $\sin \frac{3\pi}{4}$ 18) $\sin \pi$ 19) $\csc \frac{7\pi}{6}$ 20) $\sin \frac{5\pi}{3}$

21) $\cos \frac{2\pi}{3}$ 22) $\sec \frac{3\pi}{4}$ 23) $\cos \pi$ 24) $\cos \frac{7\pi}{6}$ 25) $\sec \frac{5\pi}{3}$

26) $\cot \frac{2\pi}{3}$ 27) $\tan \frac{3\pi}{4}$ 28) $\cot \pi$ 29) $\tan \frac{4\pi}{3}$ 30) $\tan \frac{11\pi}{6}$

B. Find all values of θ in terms of $(0 \leq \theta < 2\pi)$.

$$1) \sin \theta = \frac{\sqrt{3}}{2}$$

$$2) \cos \theta = -\frac{1}{2}$$

$$3) \tan \theta = -1$$

$$4) 2 \cos^2 \theta = \sqrt{3} \cos \theta$$

$$5) 2 \sin^2 \theta + \sin \theta - 1 = 0$$

C. Simplify the following expressions:

$$1) \sin \theta + \cos \theta + \sin(-\theta) + \cos(-\theta)$$

$$5) \cos^2(-\theta) - \sin \theta \sin(-\theta)$$

$$2) \frac{\tan \theta}{\sin \theta \sec \theta}$$

$$6) \csc^2 \theta - \cos^2 \theta \csc^2 \theta$$

$$3) \frac{\sin \theta}{1+\cos \theta} + \frac{\sin \theta}{1-\cos \theta}$$

$$7) \frac{1+\sin \theta}{\cos \theta} + \frac{\cos \theta}{1+\sin \theta}$$

$$4) \sin(\pi + x)$$

$$8) \cos(\frac{\pi}{2} + x)$$

D. Find all values of x in terms of π ($0 \leq x < 2\pi$)

$$1) \cos 2x + \sin x = 0$$

$$2) \sqrt{2} \sin x + \sin 2x = 0$$

$$3) \cos \frac{x}{2} = \frac{1}{2}$$

$$4) 2\sin(3x) - \sqrt{3} = 0$$

$$5) 3\tan^2 x = 1$$

$$6) 4\cos^2 x - 1 = 0.$$

E. Prove the following identities

$$1) \frac{\sin 2x}{\cos 2x + 1} = \tan x$$

$$2) \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

$$3) \csc^2 x + \sec^2 x = \csc^2 x + \sec^2 x$$

$$4) \cot x - \csc x = \frac{\cos 2x - \cos x}{\sin 2x + \sin x}$$

$$5) 2 \cos x \cos y = \cos(x + y) + \cos(x - y).$$

Answers:

A.

1. 0 2. $\frac{1}{2}$ 3. $\frac{\sqrt{2}}{2}$ 4. $\frac{\sqrt{3}}{2}$

5. 1 6. 1 7. $\frac{\sqrt{3}}{2}$ 8. $\frac{\sqrt{2}}{2}$

9. $\frac{1}{2}$ 10. 0 11. 0 12. $\frac{\sqrt{3}}{3}$

13. 1 14. $\sqrt{3}$ 15. \emptyset 16. $\frac{2\sqrt{3}}{3}$

17. $\frac{\sqrt{2}}{2}$ 18. 0 19. -2 20. $-\frac{\sqrt{3}}{2}$

21. $-\frac{1}{2}$ 22. $-\sqrt{2}$ 23. -1 24. $-\frac{\sqrt{3}}{2}$

25. 2 26. $-\frac{\sqrt{3}}{3}$ 27. -1 28. \emptyset

29. $\sqrt{3}$ 30. $-\frac{\sqrt{3}}{3}$

B.

1. $\frac{\pi}{3}, \frac{2\pi}{3}$

2. $\frac{4\pi}{3}, \frac{2\pi}{3}$

3. $\frac{3\pi}{4}, \frac{7\pi}{4}$

4. $\frac{\pi}{2}, \frac{3\pi}{2}, \dots, \frac{\pi}{6}, \dots, \frac{11\pi}{6}$

5. $\frac{3\pi}{2}, \dots, \frac{\pi}{6}, \dots, \frac{5\pi}{6}$

C.

1. $2\cos\theta$

2. 1

3. 1

4. 1

5. $2\csc\theta$

6. $2\sec\theta$

7. $-\sin x$

8. $-\sin x$

D.

1. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

2. $0, \pi, \frac{3\pi}{4}, \frac{5\pi}{4}$

3. $\frac{2\pi}{3}$

4. $\frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$

5. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

6. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$